

# Describing Charmonium Correlation Functions in Euclidean Time

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**Abstract.** We present a detailed investigation of the quark mass-dependence of charmonium correlators in Euclidean-time obtained using a potential model, as well as the comparison with results on isotropic lattice calculations performed at several lattice spacings.

## 1 Introduction

It was argued long ago that melting of quarkonia above the deconfinement transition can serve as a signature of quark gluon plasma formation in heavy ion collisions [1]. The basic idea behind this proposal was that due to color screening the potential between quark and anti-quark will not provide sufficient binding at high temperature. This problem can be formulated more rigorously in terms of quarkonium spectral functions, which can be, in principle, extracted from Euclidean-time meson correlation functions calculated on the lattice. Attempts doing this based on the Maximum Entropy Method (MEM) have been discussed over the last few years. The initial interpretation of data led to the conclusion that the  $1S$  charmonia states survive in the deconfined medium up to temperatures of about  $1.6T_c$ , with  $T_c$  being the transition temperature [2,3,4,5]. Recent analysis, however, has shown that, although MEM can be used to extract reliably quarkonium spectral functions at zero temperature, at finite temperature it has severe limitations [6].

In a recent study we explored quarkonium correlators in Euclidean time using a potential model [7]. We have shown that the temperature (in)dependence of quarkonium correlators can be explained provided that color screening melts most of the quarkonium states. The absence of bound states in a quarkonium spectral function is compensated by a large threshold enhancement, leaving the Euclidean correlators unchanged [7]. This analysis done in QCD with only heavy quarks has been extended to 2+1 flavor QCD, and was used to estimate the upper limit on the dissociation temperatures of the different quarkonium states [8]. The comparison of correlators at zero temperature was done using results from calculations done on isotropic lattices. This is because only for isotropic lattices the renormalization constants for the local meson currents are known (see discussion in Ref. [4]). The temperature-dependence of the correlators calculated in potential models has been compared against the results from anisotropic lattices. In this paper we extend our previous studies by comparing against isotropic lattice calculations at smaller lattice spacings. While in our previous calculations we had to introduce a prefactor when comparing lattice data with potential model predictions, here we do a parameter-free comparison as it is possible to use the prefactor's previously determined value.

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## 2 Charmonium Spectral Functions in Potential Model

Since the seminal paper by Matsui and Satz the problem of charmonium dissolution has been studied in potential models [10,11,12,13,14,15,16,17,18,19]. While the early studies used phenomenological potential more recent studies rely on lattice calculations of the static quark anti-quark free energy. Recently attempts to calculate quarkonium properties at finite temperature using resummed perturbation theory have been made [20,21]. Free energy calculations are done in pure gluodynamics, 3-flavor and 2-flavor QCD [22,23,24], and preliminary results are also available in the physically relevant case of one heavy strange quark and two light quarks [25] (quark masses correspond to pion mass of about 200 MeV). Since the lattice calculations of the spectral functions have severe limitations, in [15,16] it has been pointed out, that comparison between the lattice data should be done at the level of the Euclidean time correlators, for which the numerical results are much more reliable. Recent studies following this line have also been presented in Refs. [17,18,19]. However, for such comparison to be meaningful lattice artifacts in the Euclidean correlators has to be understood. Therefore, it is important to do the comparison of the potential model results with lattice calculations performed at several lattice spacings.

For heavy quarks (here we only consider charm) the spectral function can be related to the non-relativistic Green's functions

$$\sigma(\omega) = K \frac{6}{\pi} \text{Im} G^{nr}(\mathbf{r}, \mathbf{r}', E)|_{\mathbf{r}=\mathbf{r}'=0}, \quad (1)$$

$$\sigma(\omega) = K \frac{6}{\pi} \frac{1}{m_c^2} \text{Im} \nabla \cdot \nabla' G^{nr}(\mathbf{r}, \mathbf{r}', E)|_{\mathbf{r}=\mathbf{r}'=0}, \quad (2)$$

for  $S$ -wave, and  $P$ -wave charmonia, respectively. Here  $E = \omega - 2m_c$ . At leading order  $K = 1$ . Relativistic and higher order perturbative corrections will lead to a value different from unity. The non-relativistic Green's function satisfies the Schrödinger equation

$$\left[ -\frac{1}{m_c} \nabla^2 + V(r) - E \right] G^{nr}(\mathbf{r}, \mathbf{r}', E) = \delta^3(r - r'). \quad (3)$$

The numerical method for solving this equation is presented in [7]. At zero temperature we use the Cornell potential  $V(r) = -\alpha/r + \sigma r$  with parameters motivated by lattice results on static potential :  $\alpha = \pi/12$  and  $\sigma = (1.65 - \pi/12)r_0^{-2}$  (see Ref. [7] for further details.). At finite temperature we use a potential motivated by lattice results on the singlet free energy of static quark anti-quark pair and which is defined in section IV of Ref. [7]. At large energies, away from the threshold, the non-relativistic treatment is not applicable. The spectral function in this domain, however, can be calculated using perturbation theory. As in our previous work, we smoothly match the non-relativistic calculation of the spectral function to the relativistic perturbative result [7]. Euclidean time correlators  $G(\tau, T)$  at some temperature  $T$  can be calculated from the spectral functions using the integral representation

$$G_{rec}(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) K(\omega, \tau, T). \quad (4)$$

Here the integration kernel is

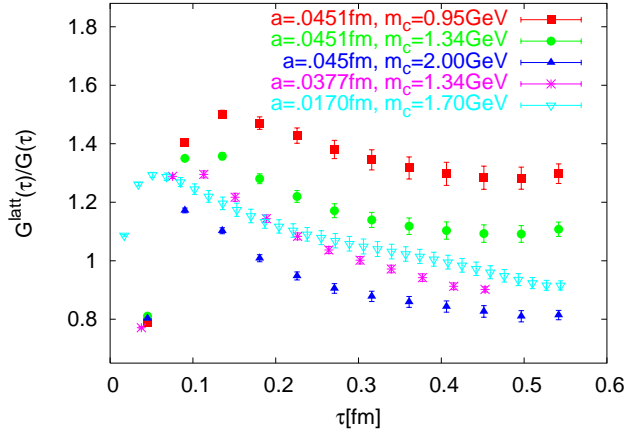
$$K(\omega, \tau, T) = \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}. \quad (5)$$

## 3 Correlators at Zero Temperature

In this section we discuss the comparison of the model calculations with zero temperature lattice data from isotropic lattices [4,9]. The lattice spacing has been fixed using the Sommer-scale  $r_0 = 0.5\text{fm}$ . Its value is slightly larger than the one used in Ref. [4], since there the string

$\beta$	$a$ [fm]	$Z_V$	$Z_P$	$M_{\eta_c}$ [GeV]	$m_c$ [GeV]
6.499	0.0451	0.975	0.847	2.622(50)	0.95
	0.0451	1.040	0.904	3.271(50)	1.34
	0.0451	1.124	1.032	4.495(10)	2.00
6.640	0.0377	1.007	0.881	3.297(270)	1.34
7.192	0.0170	0.936	0.839	4.023(52)	1.70

**Table 1.** Lattice parameters of the correlators used in the present analysis, the masses of the  $\eta_c$  obtained on isotropic lattices, and the corresponding charm quark masses used in the potential model.

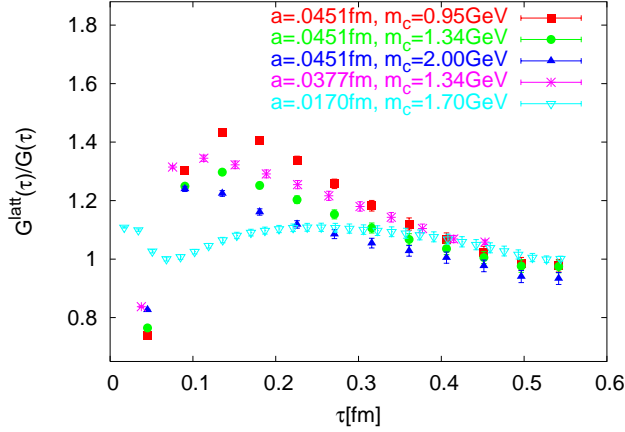


**Fig. 1.** The ratio of the pseudo-scalar correlators calculated on the lattice to the ones calculated in our model for different quark masses and lattice spacings.

tension of  $\sqrt{\sigma} = 420$  MeV has been used to set the scale. Calculations have been done at several values of the charm quark mass, but unfortunately none of them exactly at the physical value. The renormalization constants of the lattice operators has been calculated in 1-loop tadpole improved perturbation theory (see Ref. [4] for further details). In Table 1 we give the value of the gauge coupling  $\beta = 6/g^2$  used in lattice calculations, the corresponding lattice spacings, the renormalization constants  $Z_V$  and  $Z_P$  for vector and the pseudo-scalar current, as well as the estimated masses of the ground state  $\eta_c$  meson. The values of the quark masses used in potential model calculations are given here as well. All the lattice calculations have been done on a  $48^3 \times 24$  lattice. This corresponds to  $0.6T_c$  for the larger lattice spacing, and  $0.75T_c$  for the smaller lattice spacing. This is the reason why the mass of the  $\eta_c$  has a large uncertainty. For the interested reader's convenience, in Table 1 we also provide the parameters used in the analysis of Ref. [7].

We used the previously determined values of the  $K$  factors:  $K = 2.0$  for scalar and pseudo-scalar channels and  $K = 0.8$  for the vector channel [7]. In Fig. 1 we show the ratio of the pseudo-scalar correlators calculated on isotropic lattices for the parameters mentioned above and the correlator calculated in our model. Here we also show the results of the calculations at heavier quark mass  $m_c = 1.7\text{GeV}$  considered in Ref. [7]. For the smallest two lattice spacings we find a reasonable agreement for this ratio. For the coarser lattice,  $a = 0.0451\text{fm}$  our model does not seem to describe the mass dependence of the correlator very well. This could be due to the quark mass dependence of the  $K$  factor and/or lattice artifacts. Note that for  $m_c = 1.34\text{GeV}$  there is about 20% discrepancy between the results obtained at two different lattice spacings. By some tuning of the  $K$  factors, namely choosing  $K = 2.4$  and  $2.0$  for  $m_c = 0.95\text{GeV}$  and  $m_c = 2.0\text{GeV}$  a much better agreement between different lattice data can be obtained.

We also considered the vector correlators. In Fig. 2 we show the ratio of the lattice data to our model predictions. As one can see from the figure the model can capture the quark



**Fig. 2.** The ratio of the vector correlators calculated on the lattice to the ones calculated in our model for different quark masses and lattice spacings.

mass dependence of the correlators calculated on the lattice much better in this case at least for  $\tau > 0.3\text{fm}$ . At smaller Euclidean times we see significant deviations of this ratio from unity which is presumably due to lattice artifacts.

## 4 Temperature-dependence of Pseudo-scalar Correlators

In this section we study the temperature-dependence of the pseudoscalar correlator for the different quark masses considered also in the previous Section. For comparison we again use the lattice data from isotropic lattice simulations [4,9]. As customary, in order to eliminate the trivial temperature dependence in the correlators we consider the ratio  $G(\tau, T)/G_{rec}(\tau, T)$ , with

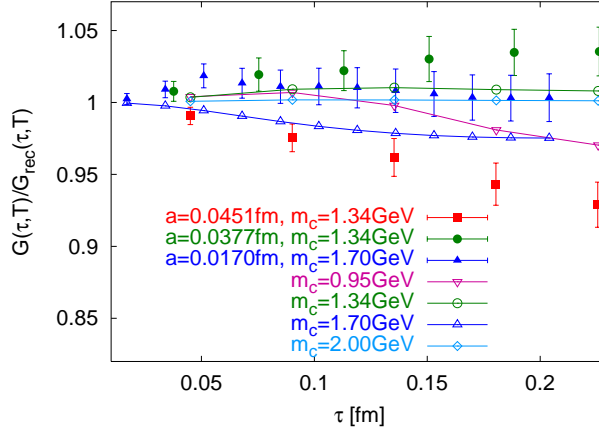
$$G_{rec}(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T=0) K(\omega, \tau, T). \quad (6)$$

The lattice data for this ratio together with potential model calculations is shown in Fig. 3 for several values of the lattice spacings at temperature  $1.5T_c$ . The lattice calculations have been performed on  $48^3 \times 10$ ,  $48^3 \times 12$  and  $64^3 \times 24$  lattices for  $a = 0.0451\text{fm}$ ,  $a = 0.0377\text{fm}$  and  $a = 0.0170\text{fm}$  respectively. The potential model predicts the ratio  $G(\tau, T)/G_{rec}(\tau, T)$  to be close to unity for all quark masses around the charm quark mass, in agreement with the analysis done in Ref. [7]. We find that the mass dependence of  $G/G_{rec}$  is about few percent. One can see from the figure that correlators calculated at lattice spacing  $a = 0.0451\text{ fm}$  and  $a = 0.0377\text{ fm}$  are quite different, even though the quark mass in these calculations is the same. From this we conclude that lattice artifacts are significant, and thus care is needed when comparing the temperature-dependence of the correlators on coarse lattices with model calculations.

The temperature dependence of the charmonium correlators is more pronounced in other channels and is mostly due to the zero mode contribution [16,26,27,28].

## 5 Conclusions

In this paper we have studied charmonium correlators at zero and at finite temperatures in potential model. We analyzed the model for several quark masses near the physical charm quark mass, and compared the results with available lattice data from isotropic lattices. At zero temperature we found a reasonably good agreement between the model calculations and the lattice data using the  $K$  factor fixed in the previous calculations done for  $M_{\eta_c} = 4.023\text{GeV}$



**Fig. 3.** The ratio  $G(\tau, T)/G_{rec}(\tau, T)$  calculated for different lattice spacings at  $1.5T_c$ .

[7]. We find that  $G/G_{rec}$  is close to unity and shows only a weak dependence on the quark mass in the quark mass region around the charm mass.

## Acknowledgments

We extend our special thank to P. Lévai, T. Biró and T. Csörgö for inviting us to, and for providing very kind hospitality during the Workshop in memoriam Prof. J. Zimányi. We further thank P. Sorensen for careful reading of the manuscript and valuable comments. We are also grateful to S. Datta for providing unpublished lattice data on charmonium correlators. This work has been supported by U.S. Department of Energy under Contract No. DE-AC02-98CH10886.

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